

# Chapter 26

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## Fluid-Particle Reactors: Design

Three factors control the design of a fluid-solid reactor; the reaction kinetics for single particles, the size distribution of solids being treated, and the flow patterns of solids and fluid in the reactor. Where the kinetics are complex and not well known, where the products of reaction form a blanketing fluid phase, where temperature within the system varies greatly from position to position, analysis of the situation becomes difficult and present design is based largely on the experiences gained by many years of operations, innovation, and small changes made on existing reactors. The blast furnace for producing iron is probably the most important industrial example of such a system.

Though some real industrial reactions may never yield to simple analysis, this should not deter us from studying idealized systems. These satisfactorily represent many real systems and in addition may be taken as the starting point for more involved analyses. Here we consider only the greatly simplified idealized systems in which the reaction kinetics, flow characteristics, and size distribution of solids are known.

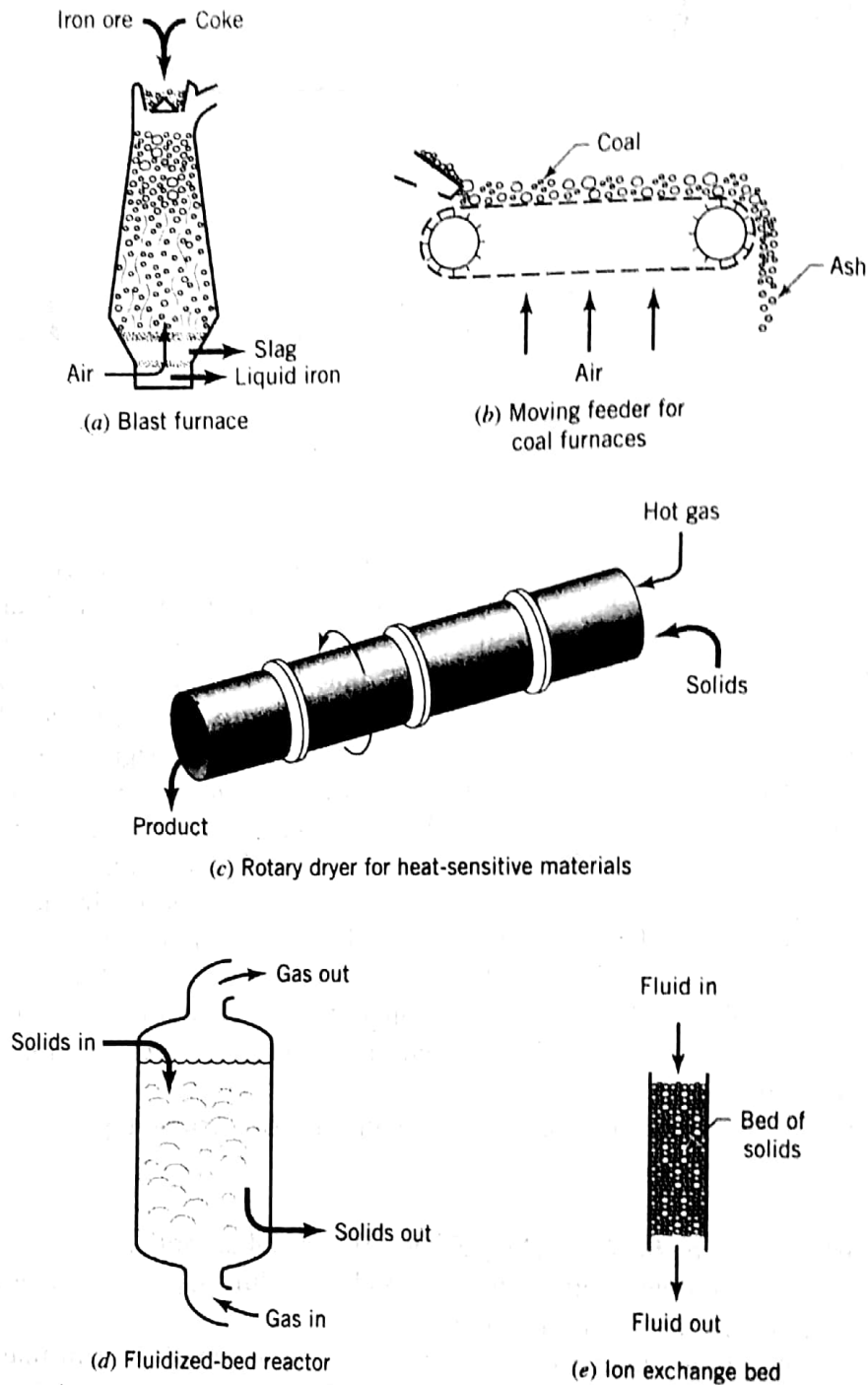
Referring to Fig. 26.1, let us discuss briefly the various types of contacting in gas-solid operations.

**Solids and Gas Both in Plug Flow.** When solids and gas pass through the reactor in plug flow, their compositions change during passage. In addition, such operations are usually nonisothermal.

The plug flow contacting of phases may be accomplished in many ways: by countercurrent flow as in blast furnaces and cement kilns [Fig. 26.1(a)], by cross-flow as in moving belt feeders for furnaces [Fig. 26.1(b)], or by cocurrent flow as in polymer driers [Fig. 26.1(c)].

**Solids in Mixed Flow.** The fluidized bed [Fig. 26.1(d)] is the best example of a reactor with mixed flow of solids. The gas flow in such reactors is difficult to characterize and often is worse than mixed flow. Because of the high heat capacity of the solids, isothermal conditions can frequently be assumed in such operations.

**Semibatch Operations.** The ion exchange column of Fig. 26.1(e) is an example of the batch treatment of solids in which the flow of fluid closely approximates



**Figure 26.1** Various contacting patterns in fluid-solid reactors: (a-d) countercurrent, crosscurrent, and cocurrent plug flow; (d) intermediate gas flow, mixed solid flow; (e) semibatch operations.

the ideal of plug flow. On the contrary, an ordinary home fireplace, another semibatch operation, has a flow which is difficult to characterize.

**Batch Operations.** The reaction and dissolution of a batch of solid in a batch of fluid, such as the acid attack of a solid, is a common example of batch operations. Analysis and design of fluid-solid systems are greatly simplified if the composi-

tion of the fluid can be considered to be uniform throughout the reactor. Since this is a reasonable approximation where fractional conversion of fluid-phase reactants is not too great, where fluid backmixing is considerable, or where solids wander about the reactor, sampling all the fluid as in fluidized beds, this assumption frequently can be used without deviating too greatly from reality. We use this assumption in the analyses that follow.

We then conclude this chapter with a brief treatment of extremely fast reactions which are representative of some combustions. Here the analysis simplifies considerably, since the kinetics do not enter the picture.

Let us now turn to a number of frequently met contacting patterns, and let us develop their performance equations, employing in every case the assumptions of uniform gas composition within the reactor.

### Particles of a Single Size, Plug Flow of Solids, Uniform Gas Composition

The contact time or reaction time needed for any specific conversion of solid is found directly from the equations of Table 25.1.

### Mixture of Particles of Different but Unchanging Sizes, Plug Flow of Solids, Uniform Gas Composition

Consider a solid feed consisting of a mixture of different-size particles. The size distribution of this feed can be represented either as a continuous distribution or as a discrete distribution. We use the latter representation because screen analysis, our way of measuring size distributions, gives discrete measurements.

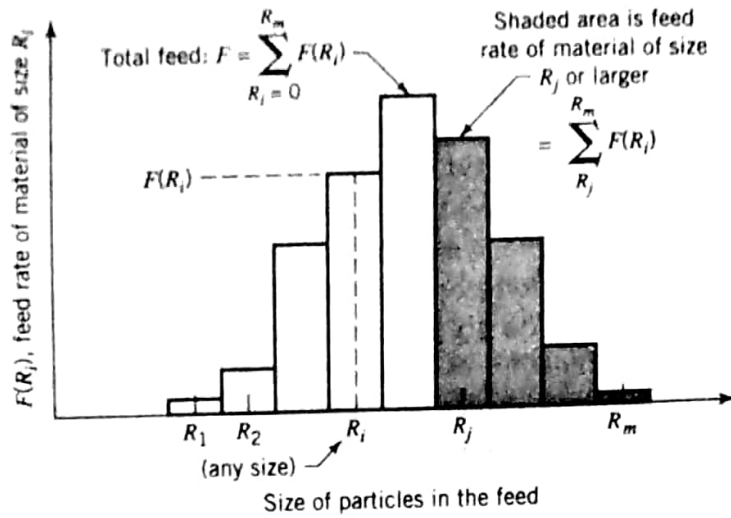
Let  $F$  be the quantity of solid being treated in unit time. Since the density of solid may change during reaction,  $F$  is defined as the volumetric feed rate of solid in the general case. Where density change of the solid is negligible,  $F$  can represent the mass feed rate of solid as well. In addition, let  $F(R_i)$  be the quantity of material of size about  $R_i$  fed to the reactor. If  $R_m$  is the largest particle size in the feed, we have for particles of unchanging size

$$F = \sum_{R_i=0}^{R_m} F(R_i), \quad \text{cm}^3/\text{sec or gm/sec}$$

Figure 26.2 shows the general characteristics of a discrete size distribution.

When in plug flow all solids stay in the reactor for the same length of time  $t_p$ . From this and the kinetics for whatever resistance controls, the conversion  $X_B(R_i)$  for any size of particle  $R_i$  can be found. Then the mean conversion  $\bar{X}_B$  of the solids leaving the reactor can be obtained by properly summing to find the overall contribution to conversion of all sizes of particles. Thus,

$$\left( \begin{array}{c} \text{mean value for} \\ \text{the fraction of} \\ \text{B unconverted} \end{array} \right) = \sum_{\text{all sizes}} \left( \begin{array}{c} \text{fraction of reactant} \\ \text{B unconverted in} \\ \text{particles of size } R_i \end{array} \right) \left( \begin{array}{c} \text{fraction of} \\ \text{feed which is} \\ \text{of size } R_i \end{array} \right) \quad (1)$$



**Figure 26.2** Representation of the feed rate of a mixture of particles.

or in symbols

$$1 - \bar{X}_B = \sum_{R(t_p=\tau)}^{R_m} [1 - X_B(R_i)] \frac{F(R_i)}{F} \quad (2)$$

where  $R(t_p = \tau)$  is the radius of the largest particle completely converted in the reactor.

Equation 2 requires some discussion. First of all, we know that a smaller particle requires a shorter time for complete conversion. Hence some of our feed particles, those smaller than  $R(t_p = \tau)$ , will be completely reacted. But if we automatically apply our conversion-time equations to these particles we can come up with  $X_B$  values greater than unity, which makes no sense physically. Thus the lower limit of the summation indicates that particles smaller than  $R(t_p = \tau)$  are completely converted and do not contribute to the fraction unconverted,  $1 - \bar{X}_B$ .

**EXAMPLE 26.1** CONVERSION OF A SIZE MIXTURE IN PLUG FLOW

A feed consisting

- 30% of 50- $\mu\text{m}$ -radius particles
- 40% of 100- $\mu\text{m}$ -radius particles
- 30% of 200- $\mu\text{m}$ -radius particles

is to be fed continuously in a thin layer onto a moving grate crosscurrent to a flow of reactant gas. For the planned operating conditions the time required for complete conversion is 5, 10, and 20 min for the three sizes of particles. Find

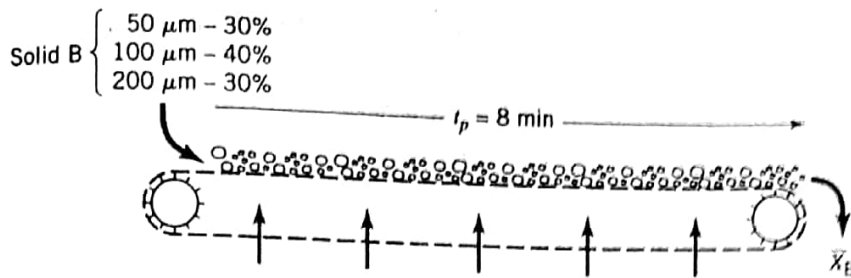


Figure E26.1

the conversion of solids on the grate for a residence time of 8 min in the reactor (see Fig. E26.1).

### SOLUTION

From the statement of the problem we may consider the solids to be in plug flow with  $t_p = 8 \text{ min}$  and the gas to be uniform in composition. Hence for a mixed feed Eq. 2 is applicable, or

$$1 - \bar{X}_B = [1 - X_B(50 \mu\text{m})] \frac{F(50 \mu\text{m})}{F} + [1 - X_B(100 \mu\text{m})] \frac{F(100 \mu\text{m})}{F} + \dots \quad (\text{i})$$

where

$$\frac{F(50 \mu\text{m})}{F} = 0.30 \quad \text{and} \quad \tau(50 \mu\text{m}) = 5 \text{ min}$$

$$\frac{F(100 \mu\text{m})}{F} = 0.40 \quad \text{and} \quad \tau(100 \mu\text{m}) = 10 \text{ min}$$

$$\frac{F(200 \mu\text{m})}{F} = 0.30 \quad \text{and} \quad \tau(200 \mu\text{m}) = 20 \text{ min}$$

Because for the three sizes of particles

$$R_1 : R_2 : R_3 = \tau_1 : \tau_2 : \tau_3,$$

we see from Eq. 25.38 that chemical reaction controls and the conversion-time characteristics for each size is given by Eq. 25.23 or

$$[1 - X_B(R_i)] = \left(1 - \frac{t_p}{\tau(R_i)}\right)^3$$

Replacing in Eq. (i) we obtain for unconverted reactant

$$1 - \bar{X}_B = 0 + \underbrace{\left(1 - \frac{8 \text{ min}}{10 \text{ min}}\right)^3}_{\text{for } R = 100 \text{ } \mu\text{m}} (0.4) + \underbrace{\left(1 - \frac{8}{20}\right)^3}_{\text{for } R = 200 \text{ } \mu\text{m}} (0.3)$$

$$= 0.0032 + 0.0648 = 0.068$$

Hence the fraction of solid converted equals 93.2%.

Note that the smallest size of particles is completely converted and does not contribute to the summation of Eq. (i).

### Mixed Flow of Particles of a Single Unchanging Size, Uniform Gas Composition

Consider the reactor of Fig. 26.1(d) with constant flow rates of both solids and gas into and out of the reactor. With the assumption of uniform gas concentration and mixed flow of solids, this model represents a fluidized-bed reactor in which there is no elutriation of fine particles.

The conversion of reactant in a single particle depends on its length of stay in the bed, and its appropriate controlling resistance is given by Eq. 25.11, 25.18, or 25.23. However, the length of stay is not the same for all the particles in the reactor; hence we must calculate a mean conversion  $\bar{X}_B$  of material. Recognizing that the solid behaves as a macrofluid, this can be done by the methods leading to Eq. 11.13. Thus, for the solids leaving the reactor

$$\left(\text{mean value for the fraction of B unconverted}\right) = \sum_{\substack{\text{particles} \\ \text{of all} \\ \text{ages}}} \left(\begin{array}{l} \text{fraction of reactant} \\ \text{unconverted for} \\ \text{particles staying in} \\ \text{the reactor for time} \\ \text{between } t \text{ and } t + dt \end{array}\right) \left(\begin{array}{l} \text{fraction of exit} \\ \text{stream which has} \\ \text{stayed in the reactor} \\ \text{for a time between} \\ t \text{ and } t + dt \end{array}\right) \quad (3)$$

or in symbols

$$1 - \bar{X}_B = \int_0^{\infty} (1 - X_B) \mathbf{E} dt, \quad X_B \leq 1 \quad (4)$$

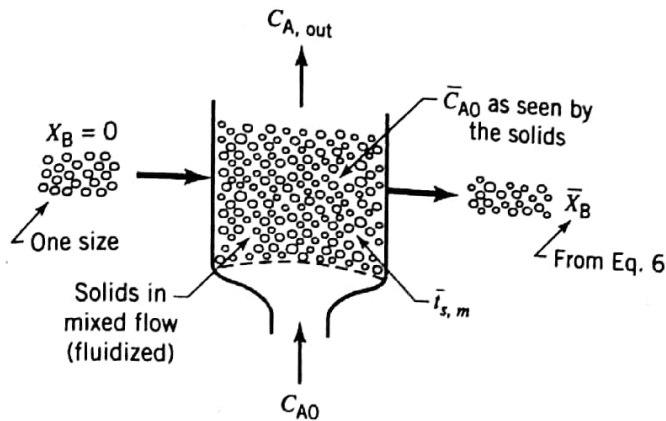
or

$$1 - \bar{X}_B = \int_0^{\tau} (1 - X_B) \mathbf{E} dt$$

where  $\mathbf{E}$  is the exit age distribution of the solids in the reactor (see Chapter 11).

For mixed flow of solids with mean residence time  $\bar{t}$  in the reactor, see Fig. 26.3, we find from Fig. 11.14 or Eq. 14.1 that

$$\mathbf{E} = \frac{e^{-t/\bar{t}}}{\bar{t}} \quad (5)$$



**Figure 26.3** Conversion of one size of solids in mixed flow.

Thus, for mixed flow of the single size of solid which is completely converted in time  $\tau$ , we obtain

$$1 - \bar{X}_B = \int_0^\tau (1 - X_{B, \text{individual particle}}) \frac{e^{-t/\bar{t}}}{\bar{t}} dt \quad (6)$$

This expression may be integrated for the various controlling resistances.

For *film resistance controlling*, Eq. 25.11 with Eq. 6 yields

$$1 - \bar{X}_B = \int_0^\tau \left(1 - \frac{t}{\tau}\right) \frac{e^{-t/\bar{t}}}{\bar{t}} dt \quad (7)$$

which on integration by parts gives

$$\bar{X}_B = \frac{\bar{t}}{\tau} (1 - e^{-\tau/\bar{t}}) \quad (8a)$$

or in equivalent expanded form, useful for large  $\bar{t}/\tau$ , thus for very high conversion

$$1 - \bar{X}_B = \frac{1}{2} \frac{\tau}{\bar{t}} - \frac{1}{3!} \left(\frac{\tau}{\bar{t}}\right)^2 + \frac{1}{4!} \left(\frac{\tau}{\bar{t}}\right)^3 - \dots \quad (8b)$$

For *chemical reaction controlling*, Eq. 25.23 replaced in Eq. 6 gives

$$1 - \bar{X}_B = \int_0^\tau \left(1 - \frac{t}{\tau}\right)^3 \frac{e^{-t/\bar{t}}}{\bar{t}} dt \quad (9)$$

Integrating by parts using the recursion formula, found in any table of integrals, we obtain

$$\bar{X}_B = 3 \frac{\bar{t}}{\tau} - 6 \left(\frac{\bar{t}}{\tau}\right)^2 + 6 \left(\frac{\bar{t}}{\tau}\right)^3 (1 - e^{-\tau/\bar{t}}) \quad (10a)$$

or in equivalent form, useful for large  $\bar{t}/\tau$ , or for very high conversion

$$1 - \bar{X}_B = \frac{1}{4} \frac{\tau}{\bar{t}} - \frac{1}{20} \left(\frac{\tau}{\bar{t}}\right)^2 + \frac{1}{120} \left(\frac{\tau}{\bar{t}}\right)^3 - \dots \quad (10b)$$

For *ash resistance controlling* replacement of Eq. 25.18 in Eq. 6 followed by integration leads to a cumbersome expression which on expansion yields [see Yagi and Kunii (1961)]

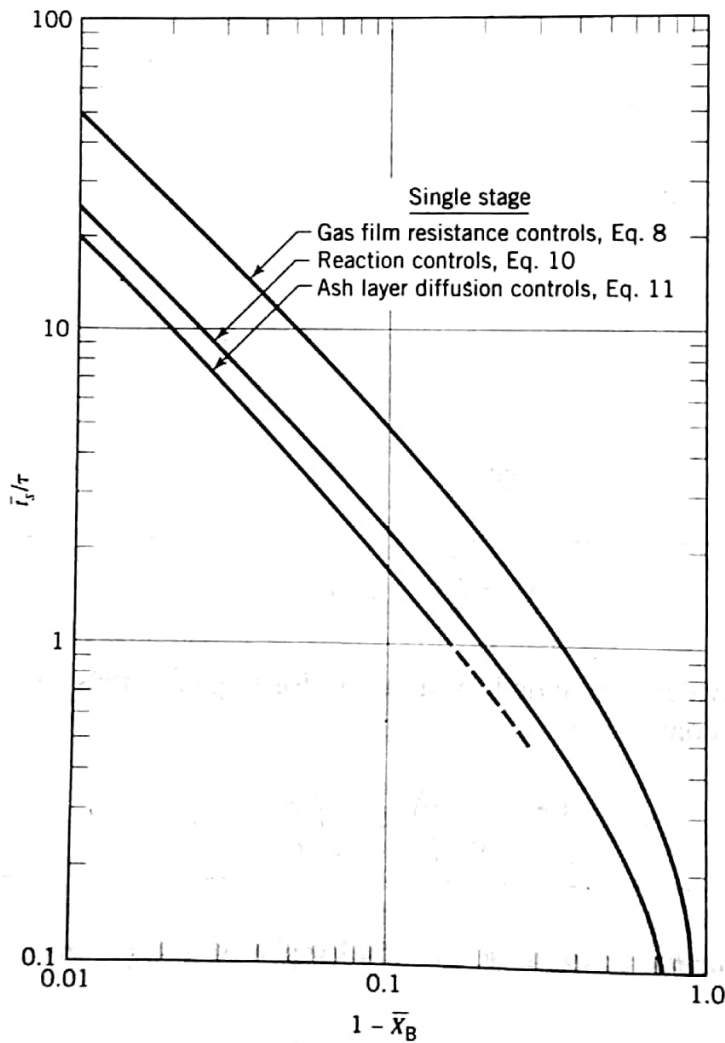
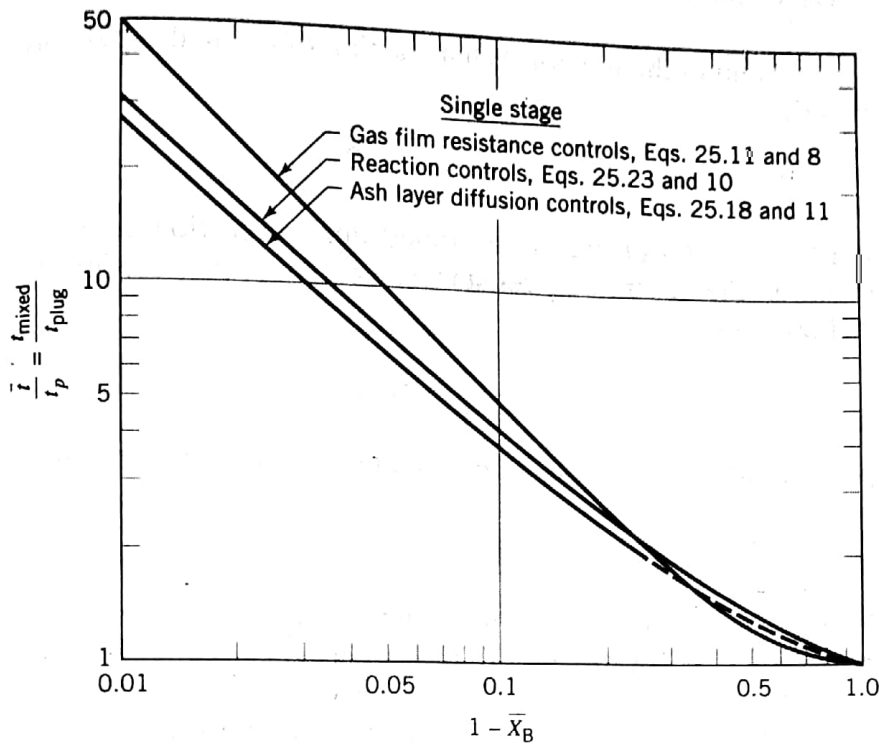


Figure 26.4 Mean conversion versus mean residence time in mixed flow reactors, single size of solid.





**Figure 26.5** Comparison of holding times needed to effect a given conversion for mixed flow and plug flow of a single size of solid.

$$1 - \bar{X}_B = \frac{1}{5} \frac{\tau}{t} - \frac{19}{420} \left( \frac{\tau}{t} \right)^2 + \frac{41}{4620} \left( \frac{\tau}{t} \right)^3 - 0.00149 \left( \frac{\tau}{t} \right)^4 + \dots \quad (11)$$

Figures 26.4 and 26.5 present these results for solids in mixed flow in convenient graphical form. Figure 26.5 shows clearly that at high conversion the mixed flow reactor requires a much larger holding time for solids than does a plug flow reactor.

Extension to multistage operations is not difficult; see Levenspiel (1996, p. 52.14) or Kunii and Levenspiel (1991).

### EXAMPLE 26.2 CONVERSION OF A SINGLE-SIZED FEED IN A MIXED FLOW REACTOR

Yagi et al. (1951) roasted pyrrhotite (iron sulfide) particles dispersed in asbestos fibers and found that the time for complete conversion was related to particle size as follows:

$$\tau \propto R^{1.5}$$

Particles remained as hard solids of unchanging size during reaction.

A fluidized-bed reactor is planned to convert pyrrhotite ore to the corresponding oxide. The feed is to be uniform in size,  $\tau = 20$  min, with mean residence

time  $\bar{t} = 60$  min in the reactor. What fraction of original sulfide ore remains unconverted?

### SOLUTION

Since a hard product material is formed during reaction, film diffusion can be ruled out as the controlling resistance. For chemical reaction controlling Eq. 25.38 shows that

$$\tau \propto R$$

whereas for ash layer diffusion controlling Eq. 25.37 shows that

$$\tau \propto R^2$$

As the experimentally found diameter dependency lies between these two values, it is reasonable to expect that both these mechanisms offer resistance to conversion. Using in turn ash diffusion and chemical reaction as the controlling resistance should then give the upper and lower bound to the conversion expected.

The solids in a fluidized bed approximate mixed flow; hence, for chemical reaction controlling, Eq. 10, with  $\tau/\bar{t} = 20 \text{ min}/60 \text{ min} = \frac{1}{3}$ , gives

$$1 - \bar{X}_B = \frac{1}{4} \left(\frac{1}{3}\right) - \frac{1}{20} \left(\frac{1}{3}\right)^2 + \frac{1}{120} \left(\frac{1}{3}\right)^3 - \dots = 0.078$$

For ash layer diffusion controlling Eq. 11 gives

$$1 - \bar{X}_B = \frac{1}{5} \left(\frac{1}{3}\right) - \frac{19}{420} \left(\frac{1}{3}\right)^2 + \frac{41}{4620} \left(\frac{1}{3}\right)^3 - \dots = 0.062$$

Hence the fraction of sulfide remaining is between 6.2% and 7.8%, or on averaging

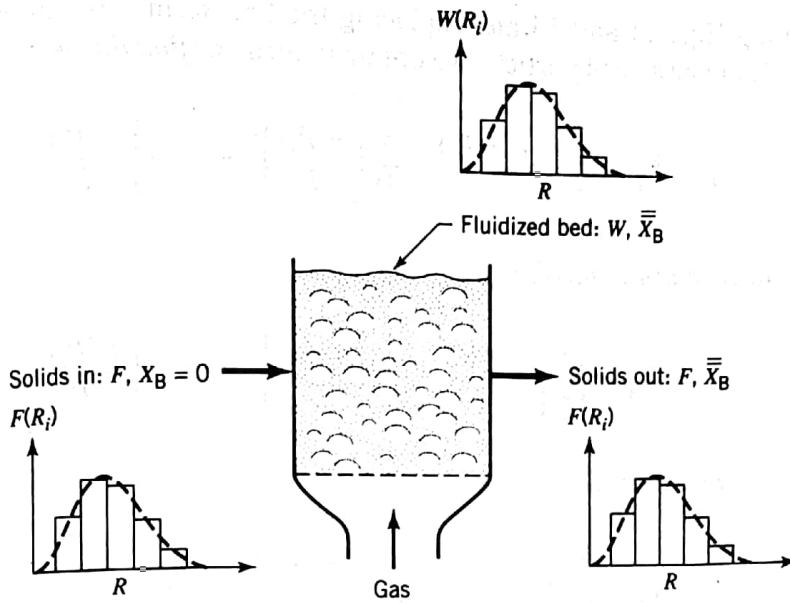
$$\underline{\underline{1 - \bar{X}_B \cong 0.07}}, \quad \text{or} \quad \underline{\underline{7.0\%}}$$

### Mixed Flow of a Size Mixture of Particles of Unchanging Size, Uniform Gas Composition

Often a range of particle sizes is used as feed to a mixed flow reactor. For such a feed and a single-exit stream (no elutriation of the fines) the methods leading to Eqs. 2 and 6, when combined, should yield the required conversion.

Consider the reactor shown in Fig. 26.6. Since the exit stream is representative of the bed conditions, the size distributions of the bed as well as the feed and exit streams are all alike, or

$$\frac{F(R_i)}{F} = \frac{W(R_i)}{W} \quad (12)$$



**Figure 26.6** Fluidized bed with single exit stream treating a size mixture of solids. Note that the size distribution of the flow streams and the bed are all the same.

where  $W$  is the quantity of material in the reactor and where  $W(R_i)$  is the quantity of material of size  $R_i$  in the reactor. In addition, for this flow the mean residence time  $\bar{t}(R_i)$  of material of any size  $R_i$  is equal to the mean residence time of solid in the bed, or

$$\bar{t} = \bar{t}(R_i) = \frac{W}{F} = \frac{\text{(weight of all solids in the reactor)}}{\text{(feed rate of all solids to the reactor)}} \quad (13)$$

Letting  $\bar{X}_B(R_i)$  be the mean conversion of particles of size  $R_i$  in the bed, we have from Eq. 6

$$1 - \bar{X}_B(R_i) = \int_0^{\tau(R_i)} [1 - X_B(R_i)] \frac{e^{-t/\bar{t}}}{\bar{t}} dt \quad (14)$$

However, the feed consists of particles of different sizes; hence the overall mean of B unconverted in all these sizes is

$$\left( \begin{array}{l} \text{mean value for} \\ \text{fraction of B} \\ \text{unconverted} \end{array} \right) = \sum_{\text{all sizes}} \left( \begin{array}{l} \text{fraction unconverted} \\ \text{in particles of size } R_i \end{array} \right) \left( \begin{array}{l} \text{fraction of exit or} \\ \text{entering stream consisting} \\ \text{of particles of size } R_i \end{array} \right) \quad (15)$$

or in symbols

$$1 - \bar{X}_B = \sum_{R=0}^{R_m} [1 - X_B(R_i)] \frac{F(R_i)}{F}$$

Combining Eqs. 14 and 15 and replacing the first term expression with Eqs. 8, 10, or 11 for each size of particle, we obtain in turn, for *film diffusion controlling*,

$$1 - \bar{X}_B = \sum_{R_m} \left\{ \frac{1}{2!} \frac{\tau(R_i)}{\bar{t}} - \frac{1}{3!} \left[ \frac{\tau(R_i)}{\bar{t}} \right]^2 + \dots \right\} \frac{F(R_i)}{F} \quad (16)$$

for *chemical reaction controlling*,

$$1 - \bar{X}_B = \sum_{R_m} \left\{ \frac{1}{4} \frac{\tau(R_i)}{\bar{t}} - \frac{1}{20} \left[ \frac{\tau(R_i)}{\bar{t}} \right]^2 + \dots \right\} \frac{F(R_i)}{F} \quad (17)$$

for *ash diffusion controlling*,

$$1 - \bar{X}_B = \sum_{R_m} \left\{ \frac{1}{5} \frac{\tau(R_i)}{\bar{t}} - \frac{19}{420} \left[ \frac{\tau(R_i)}{\bar{t}} \right]^2 + \dots \right\} \frac{F(R_i)}{F} \quad (18)$$

where  $\tau(R_i)$  is the time for complete reaction of particles of size  $R_i$ . The following example illustrates the use of these expressions.

### EXAMPLE 26.3

### CONVERSION OF A FEED MIXTURE IN A MIXED FLOW REACTOR

A feed consisting

- 30% of 50- $\mu\text{m}$ -radius particles
- 40% of 100- $\mu\text{m}$ -radius particles
- 30% of 200- $\mu\text{m}$ -radius particles

is to be reacted in a fluidized-bed steady-state flow reactor constructed from a vertical 2-m long 20-cm ID pipe. The fluidizing gas is the gas-phase reactant, and at the planned operating conditions the time required for complete conversion is 5, 10, and 20 min for the three sizes of feed. Find the conversion of solids in the reactor for a feed rate of 1 kg solids/min if the bed contains 10 kg solids.

#### Additional Information:

The solids are hard and unchanged in size and weight during reaction. A cyclone separator is used to separate and return to the bed any solids that may be entrained by the gas. The change in gas-phase composition in the bed is small.

#### SOLUTION

From the statement of the problem we may consider the solids to be in mixed flow. For a feed mixture Eq. 15 is applicable, and since chemical reaction controls (see Example 26.1), this equation reduces to Eq. 17, where from the problem

statement

$$F = 1000 \text{ gm/min} \quad \bar{t} = \frac{W}{F} = \frac{10\,000 \text{ gm}}{1000 \text{ gm/min}} = 10 \text{ min}$$

$$W = 10\,000 \text{ gm}$$

$$\begin{array}{lll} F(50 \mu\text{m}) = 300 \text{ gm/min} & \text{and} & \tau(50 \mu\text{m}) = 5 \text{ min} \\ F(100 \mu\text{m}) = 400 \text{ gm/min} & \text{and} & \tau(100 \mu\text{m}) = 10 \text{ min} \\ F(200 \mu\text{m}) = 300 \text{ gm/min} & \text{and} & \tau(200 \mu\text{m}) = 20 \text{ min} \end{array}$$

Replacing in Eq. 17 we obtain

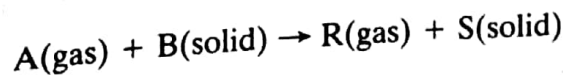
$$\begin{aligned} 1 - \bar{X}_B &= \left[ \frac{1}{4} \frac{5 \text{ min}}{10 \text{ min}} - \frac{1}{20} \left( \frac{5}{10} \right)^2 + \dots \right] \frac{300 \text{ gm/min}}{1000 \text{ gm/min}} \\ &\quad \text{for } R = 50 \mu\text{m} \\ &+ \left[ \frac{1}{4} \left( \frac{10 \text{ min}}{10 \text{ min}} \right) - \frac{1}{20} \left( \frac{10}{10} \right)^2 + \dots \right] \frac{400}{1000} \\ &\quad \text{for } R = 100 \mu\text{m} \\ &+ \left[ \frac{1}{4} \left( \frac{20 \text{ min}}{10 \text{ min}} \right) - \frac{1}{20} \left( \frac{20}{10} \right)^2 + \dots \right] \frac{300}{1000} \\ &\quad \text{for } R = 200 \mu\text{m} \\ &= \left( \frac{1}{8} - \frac{1}{80} + \dots \right) \frac{3}{10} + \left( \frac{1}{4} - \frac{1}{20} + \frac{1}{120} - \dots \right) \frac{4}{10} \\ &\quad + \left( \frac{1}{2} - \frac{1}{5} + \frac{1}{15} - \frac{2}{10} + \dots \right) \frac{3}{10} \\ &= 0.034 + 0.083 + 0.105 = 0.222 \end{aligned}$$

The mean conversion of solids is then

$$\bar{X}_B = 77.8\%$$

### FINDING THE SIZE OF A FLUIDIZED BED

In a gas-phase environment, particles of B are converted to solid product as follows:



Reaction proceeds according to the shrinking core model with reaction control and with time for complete conversion of particles of 1 hr.

A fluidized bed is to be designed to treat 1 ton/hr of solids to 90% conversion using a stoichiometric feed rate of A, fed at  $C_{A0}$ . Find the weight of solids in

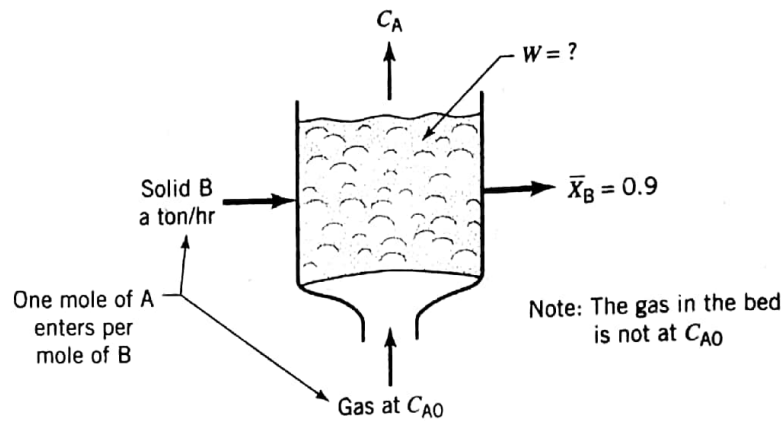


Figure E26.4

the reactor if gas is assumed to be in mixed flow. Note that the gas in the reactor is not at  $C_{A0}$ . Figure E26.4 sketches this problem.

### SOLUTION

In a  $C_{A0}$  environment with reaction controlling

$$\tau = \frac{\rho_B R}{k_s C_{A0}} = 1 \text{ hr}$$

and in any other environment  $\tau \propto \frac{1}{C_{A0}}$

Now for equal stoichiometric feed  $X_A = \bar{X}_B$ . Thus, the leaving gas is at  $0.1 C_{A0}$ . Since the gas is in mixed flow, the solids see this exit gas, or  $\tau = 10$  hr.

From Eq. 10 let us find  $\tau/\bar{t}$  which gives  $\bar{X}_B = 0.9$ . Thus, solve

$$1 - \bar{X}_B = 0.1 = \frac{1}{4} \left( \frac{\tau}{\bar{t}} \right) - \frac{1}{20} \left( \frac{\tau}{\bar{t}} \right)^2 + \dots$$

Solving by trial and error we find

$$\tau/\bar{t} = 0.435, \quad \text{or} \quad \bar{t} = \frac{W}{F_{B0}} = 23 \text{ hr}$$

Thus, the needed weight of bed is

$$W = \bar{t} F_{B0} = 23(1) = \underline{\underline{23 \text{ tons}}}$$

## Instantaneous Reaction

When reaction between gas and solid is fast enough so that any volume element of reactor contains only one or other of the two reactants, but not both, then we may consider reaction to be instantaneous. This extreme is approached in the high temperature combustion of finely divided solids.

In this situation prediction of the performance of the reactor is straightforward and is dependent only on the stoichiometry of the reaction. The kinetics do not enter the picture. Let us illustrate this behavior with the following ideal contacting patterns.

**Batch Solids.** Figure 26.7 shows two situations, one which represents a packed bed, the other a fluidized bed with no bypassing of gas in the form of large gas bubbles. In both cases the leaving gas is completely converted and remains that way so long as solid reactant is still present in the bed. As soon as the solids are all consumed, and this occurs the instant the stoichiometric quantity of gas has been added, then the conversion of gas drops to zero.

**Countercurrent Plug Flow of Gas and Solids.** Since only one or other reactant can be present at any level in the bed, there will be a sharp reaction plane where the reactants meet. This will occur either at one end or the other of the reactor depending on which feed stream is in excess of stoichiometric. Assuming that each 100 moles of solid combine with 100 moles of gas, Figs. 26.8a and b show what happens when we feed a little less gas than stoichiometric and a little more than stoichiometric.

We may wish reaction to occur in the center of the bed so that both ends can be used as heat exchange regions to heat up reactants. This can be done by matching the gas and solids flow rates; however, this is inherently an unstable system and requires proper control. A second alternative, shown in Fig. 26.8c,

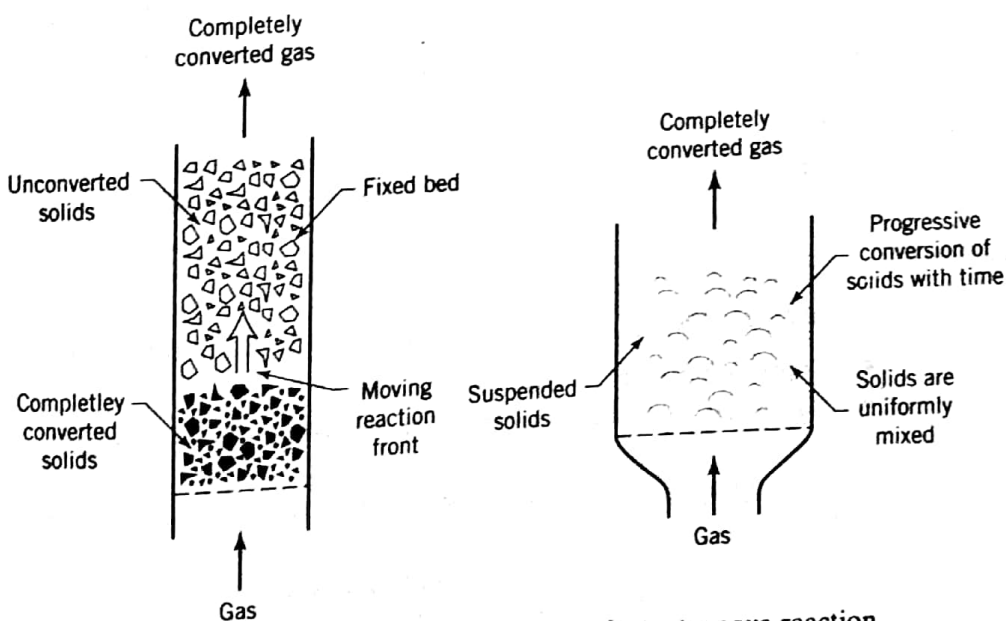
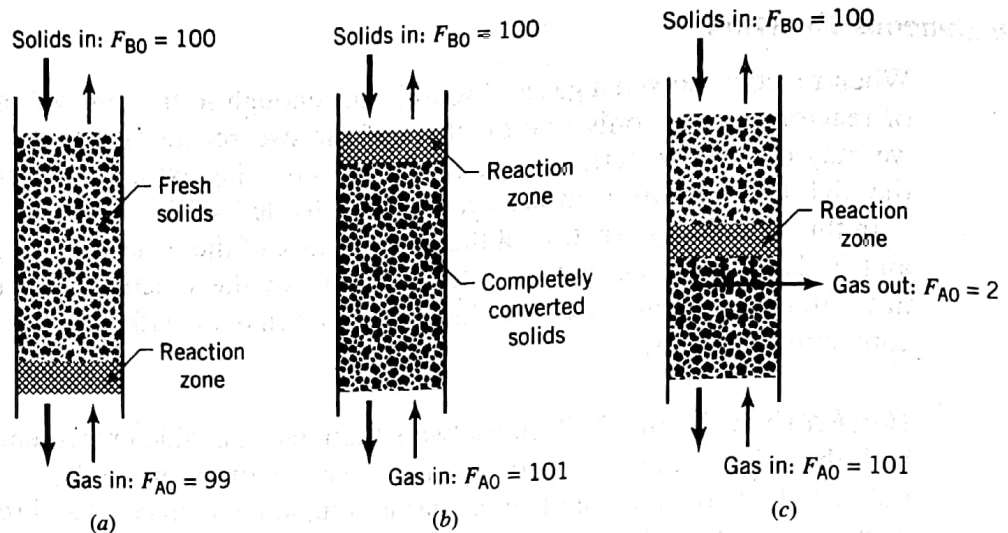


Figure 26.7 A batch of solids contacted with gas; instantaneous reaction.



**Figure 26.8** In countercurrent flow the location of the reaction zone depends on which component is in excess of stoichiometric.

introduces a slight excess of gas at the bottom of the bed, and then removes a bit more than this excess at the point where reaction is to occur.

Moving bed reactors for oil recovery from shale is one example of this kind of operation. Another somewhat analogous operation is the multistage counterflow reactor, and the four- or five-stage fluidized calciner is a good example of this. In all these operations the efficiency of heat utilization is the main concern.

**Cocurrent and Crosscurrent Plug Flow of Gas and Solids.** In cocurrent flow, shown in Fig. 26.9a, all reaction occurs at the feed end, and this represents a poor method of contacting with regard to efficiency of heat utilization and pre-heating of entering materials.

For crosscurrent flow, shown in Fig. 26.9b, there will be a definite reaction plane in the solids whose angle depends solely on the stoichiometry and the relative feed rate of reactants. In practice, heat transfer characteristics may somewhat modify the angle of this plane.

**Mixed Flow of Solids and Gas.** Again in the ideal situation either gas or solid will be completely converted in the reactor depending on which stream is in excess.

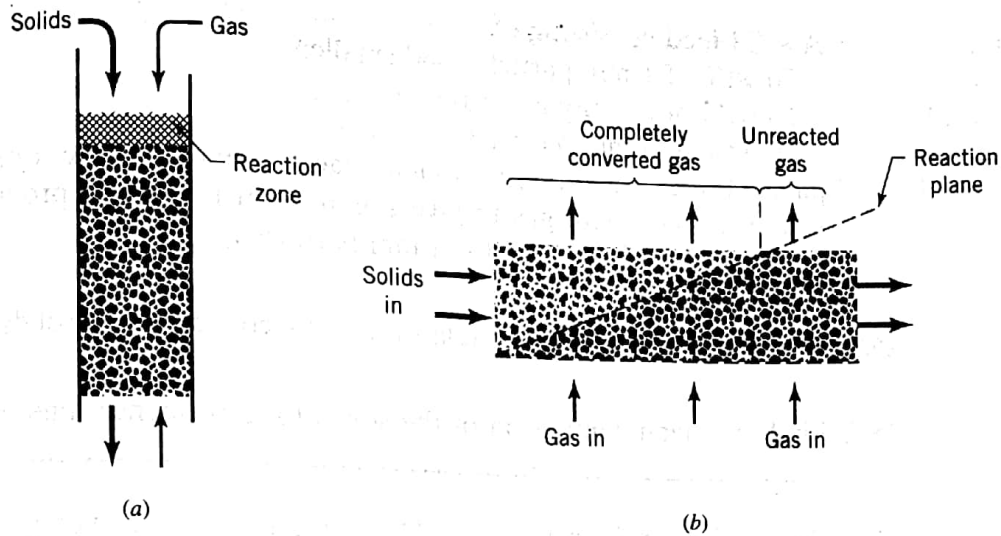
## Extensions

Modifications and extensions of the methods presented here, for example,

- to more complicated particle kinetics
- to growing and shrinking particles in single reactors and in solid circulation systems
- to changing gas composition in a single reactor and from stage to stage in multistage operations
- to deviations from ideal plug and mixed flow
- to elutriation of fines from a reactor

are treated elsewhere: see Kunii and Levenspiel (1991), and Levenspiel (1996).





**Figure 26.9** Cocurrent and crosscurrent contacting of gas-solids; instantaneous reaction.

**Arbitrary Flow of Solids.** Since particles flow as macrofluids, their mean conversion is given by Eq. 11.13. Thus with any RTD and with known gas composition we have

$$1 - \bar{X}_B = \int_0^{\tau} (1 - X_B)_{\text{single particle}} E_{\text{solid stream}} dt \quad (19)$$

where  $1 - X_B$  is given by Eq. 25.23 for SCM/reaction control, and by Eq. 25.18 for SCM/ash diffusion control.

### REFERENCES

Kunii, D., and Levenspiel, O., *Fluidization Engineering*, Second edition, Butterworth, Boston, MA, 1991.  
 Levenspiel, O., *Chemical Reactor Omnibook*, OSU Bookstores, Corvallis, OR 97339, 1996.  
 Yagi, S., and Kunii, D., *Chem. Eng. Sci.*, **16**, 364, 372, 380 (1961).  
 \_\_\_\_\_, Takagi, K., and Shimoyama, S., *J. Chem. Soc. (Japan), Ind. Chem. Sec.*, **54**, 1 (1951).

### PROBLEMS

A stream of particles of one size are 80% converted (SCM/ash diffusion control, uniform gas environment) on passing through a reactor. If the reactor is made twice the size but with the same gas environment, same feed rate, and same flow pattern of solids, what would be the conversion of solids? The solids are in

26.1. plug flow.

26.2. mixed flow.

A solid feed consisting of  
 20 wt% of 1-mm particles and smaller  
 30 wt% of 2-mm particles  
 50 wt% of 4-mm particles  
 passes through a rotating tubular reactor somewhat like a cement kiln  
 where it reacts with gas to give a hard nonfriable solid product (SCM/  
 reaction control,  $\tau = 4$  h for 4-mm particles).

26.3. Find the residence time needed for 100% conversion of solids.

26.4. Find the mean conversion of the solids for a residence time of 15 min.

26.5. Particles of uniform size are 60% converted on the average (shrinking core model with reaction controlling) when flowing through a single fluidized bed. If the reactor is made twice as large but contains the same amount of solids and with the same gas environment what would be the conversion of solids?

26.6. Solids of unchanging size,  $R = 0.3$  mm, are reacted with gas in a steady flow bench scale fluidized reactor with the following result.

$$F_0 = 10 \text{ gm/sec}, \quad W = 1000 \text{ gm}, \quad \bar{X}_B = 0.75$$

Also, the conversion is strongly temperature-sensitive suggesting that the reaction step is rate-controlling. Design a commercial sized fluidized bed reactor (find  $W$ ) to treat 4 metric tons/hr of solid feed of size  $R = 0.3$  mm to 98% conversion.

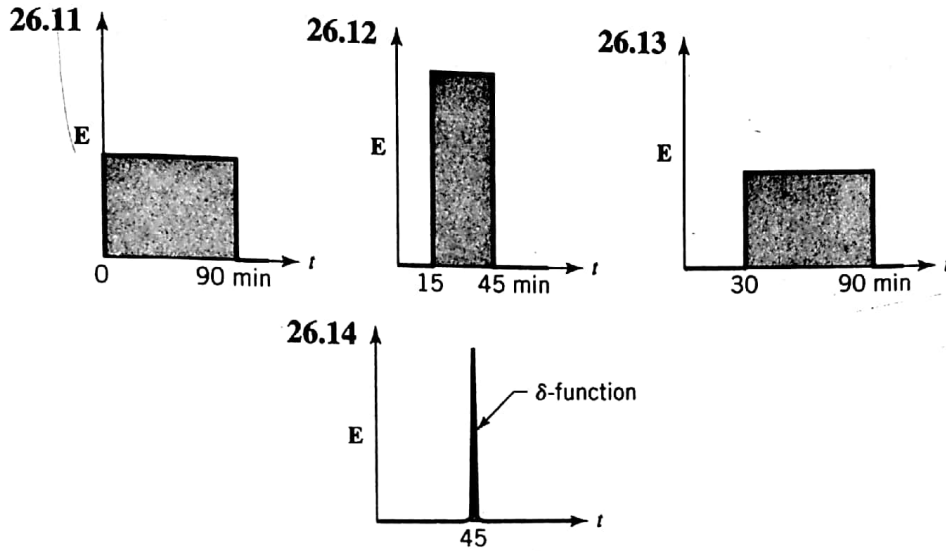
26.7. Solve Example 26.3 with the following modification: the kinetics of the reaction is ash diffusion controlled with  $\tau(R = 100 \mu\text{m}) = 10$  min.

26.8. Repeat Example 26.4 if twice the stoichiometric ratio of gas to solid, still at  $C_{A0}$ , is fed to the reactor.

26.9. Repeat Example 26.4 if the gas is assumed to pass in plug flow through the reactor.

26.10. Consider the following process for converting waste shredded fibers into a useful product. Fibers and fluid are fed continuously into a mixed flow reactor where they react according to the shrinking core model with the reaction step as rate controlling. Develop the performance expression for this operation as a function of the pertinent parameters and ignore elutriation.

Hydrogen sulfide is removed from coal gas by passing the gas through a moving bed of iron oxide particles. In the coal gas environment (consider uniform) the solids are converted from  $\text{Fe}_2\text{O}_3$  to  $\text{FeS}$  by the SCM/reaction control,  $\tau = 1$  hr. Find the fractional conversion of oxide to iron sulfide if the RTD of solids in the reactor is approximated by the  $E$  curves of Figs. P26.11–P26.14.



Figures P26.11–P26.14